Survival Kit

Filters





A filter is an electronic circuit (quadripole) whose behavior depends on the frequency of the signals. It is therefore a circuit that must contain at least one capacitor or one coil.

Rappels :

- Capacitor : The complex impedance of this dipole is $\underline{Z_C} = \frac{1}{jC\omega}$. So we have :
  - $\left|\underline{Z_C}\right| = \frac{1}{C\omega} \underset{\omega \to 0}{\longrightarrow} \infty$: The capacitor behaves like a dipole with infinite resistance at very low frequencies. It is therefore equivalent to an open switch.
  - $\left|\underline{Z_C}\right| = \frac{1}{C\omega} \underset{\omega \to \infty}{\longrightarrow} 0$: The capacitor behaves like a dipole with no resistance at very high frequencies. It is therefore equivalent to a wire.
- Coil : The complex impedance of this dipole is $\underline{Z_L} = jL\omega$. So we have :
  - $\left|\underline{Z_L}\right| = L\omega \underset{\omega \to 0}{\longrightarrow} 0$: The coil behaves like a dipole with no resistance at very low frequencies. It is therefore equivalent to a wire.
  - $\left|\underline{Z_L}\right| = L\omega \underset{\omega \to \infty}{\longrightarrow} \infty$: The coil behaves like a dipole with infinite resistance at very high frequencies. It is therefore equivalent to an open switch.

To study a filter, we examine its **transfer function** $\underline{H}(\omega)$ which is the quotient of the complex amplitude of the output voltage $\underline{V_S}$ divided by the complex amplitude of the input voltage $\underline{V_E}$. The module of this transfer function is called the **gain** and is denoted by $G(\omega)$. Its argument represents the phase shift of the output voltage relative to the input voltage.

So we have :

$$\begin{cases} \underline{H}(\omega) = \dfrac{\underline{V_S}}{\underline{V_E}} \\ G(\omega) = \left|\underline{H}(\omega)\right| = \left|\dfrac{\underline{V_S}}{\underline{V_E}}\right| \\ \arg\left(\underline{H}(\omega)\right) = \arg\left(\underline{V_S}\right) - \arg\left(\underline{V_E}\right) \end{cases}$$

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Note: As the gain of a filter can vary significantly, it is more useful to work on its **gain in decibels**. $G_{dB}(\omega) = 20.\log\big(G(\omega)\big)$

There are 4 types of filters:

- **Low pass** when $G(\omega)$ or $G_{dB}(\omega)$ is a decreasing function..
- **High pass** when $G(\omega)$ or $G_{dB}(\omega)$ is an increasing function.
- **Band pass** when $G(\omega)$ or $G_{dB}(\omega)$ is a function that is first increasing and then decreasing.
- **Notch (band-reject)** when $G(\omega)$ or $G_{dB}(\omega)$ is a function that is first decreasing and then increasing.

The **order of a filte**r is defined by the highest degree of the polynomial that allows the transfer function to be written. In practical terms, it corresponds to the number of non-resistive passive components, i.e., the number of capacitors and coils: a filter containing only one capacitor will therefore be a first-order filter, while a filter containing one capacitor and two coils will be a third-order filter